Assignment 1 of MATP6600/ISYE6780

(Due on Sep-21-2018 in class)

Exercises 2.7, 2.12, 2.52, 2.54 in the textbook "Nonlinear programming: theory and algorithms, 3rd Ed. by Bazaraa-Sherali-Shetty".

Additional problems

1. Let S be a nonempty closed set. Give an example to show that $\text{conv}(S)$ is not necessarily closed. Prove that if S is a nonempty compact set, then $\text{conv}(S)$ is closed.
2. Let $\mathbf{A} \in \mathbb{R}^{n\times n}$ be a symmetric matrix, $\mathbf{b} \in \mathbb{R}^n$, and $c \in \mathbb{R}$. The set S is defined as

$$S = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b}^\top \mathbf{x} + c \leq 0\}$$

(i) Show that S is convex if $\mathbf{A}$ is positive semidefinite (under the convention that an empty set is convex)

(ii) Show that the intersection of S and the hyperplane defined by $\mathbf{a}^\top \mathbf{x} + d = 0$ where $\mathbf{a} \neq \mathbf{0}$ is convex if $\mathbf{A} + \lambda \mathbf{a}\mathbf{a}^\top$ is positive semidefinite for some $\lambda \in \mathbb{R}$.