Alternating direction method of multipliers for robust PCA

(Due in class April-27-2018)

1 Robust principal component analysis

Let X be composed of a sparse matrix S and a low-rank matrix L. The robust PCA aims at finding S and L, given X. Using ℓ_1 norm to promote sparsity and nuclear norm to promote low-rankness, robust PCA can be modeled as

$$\min_{L,S} \|L\|_* + \lambda\|S\|_1, \text{ s.t. } L + S = X. \tag{1}$$

Here, $\|L\|_*$ denotes the nuclear norm of L and equals the summation of its singular values, and $\|S\|_1 = \sum_{i,j} |s_{ij}|$.

2 Requirements

1. In the previous homework, you were asked to apply alternating minimization (AltMin) to (a sequence of) the penalized problem

$$\min_{L,S} \|L\|_* + \lambda\|S\|_1 + \frac{\beta}{2}\|L + S - X\|_F^2, \tag{2}$$

   where $\beta > 0$ is the penalty parameter. You may notice that β has to be very big such that the solution of (2) is close to that of (1). In this homework, apply the alternating direction method of multipliers (ADMM) and the symmetric ADMM directly to (1). There is also a penalty parameter in ADMM. Tune that parameter and report the convergence behavior of the methods with different penalty parameters.

2. Compare the two solvers to your penalty method on the provided Escalator video Dataset. Report what you observe.